HOW A BALL FALLS

SIEMENS STEM DAY | Science Technology Engineering Math

STEM Category

Math

CAREER PATH

IT

TOPIC

Linear, Quadratic, & Exponential Models

OVERVIEW

In this activity, students work in teams to develop a method for creating a mathematical model of the path of a ball. The teams measure the height from which a ball is thrown, and the time taken for the ball to reach the ground. The teams video the trajectory of the ball, and then use their data to create a model of the trajectory by factoring a quadratic equation. The teams use their equations to generate a graph that models the parabola. Teams present their results for peer evaluation of the accuracy of their mathematical models compared with their video observations.

STEM LESSON FOCUS

Engineering Design Cycle	21st Century Skills
• Creating or Protoyping	• Critical Thinking

OBJECTIVES

Students will be able to:

- Model the trajectory of a projectile.
- Evaluate the validity of a model using a real-world activity.

HOW A BALL FALLS

MATERIALS

- Several different kinds of balls (baseball, tennis ball, ping-pong ball, etc.)
- Cell-phone (with video capability)
- Tape measure
- Timer
- Roll of paper
- Marker pen
- Duct tape

HAVE YOU EVER WONDERED…

How to predict the path of a baseball or similar thrown object? How programmers create a video game that includes projectiles?

MAKE CONNECTIONS!

How does this connect to students?	How does this connect to careers?	How does this connect to our world?
Students will see that sporting activities they enjoy can be studied using mathematical models.	Sports scientists help athletes to improve performance. Evaluating and modeling gives insight in how improving specific actions can contribute to performance. Video game designers and programmers must understand the physics of moving objects. Modeling the parabola of a projectile demonstrates the	People in a wide range of industries and disciplines use quadratic functions. A business owner can use quadratic equations to predict profits and losses. Conservationists use quadratic equations to model changes in populations of endangered species.

HOW A BALL FALLS

	mathematical foundation for designing such games. Space scientist—The physics of motion is a core skill for space scientists. Whether scientists work on rocket trajectories or the motion of satellites, they need to know how to use quadratic equations.	

BLUEPRINT FOR DISCOVERY

Blueprint for Discovery

1. Provide (or ask students to bring in) a variety of commonly available spherical balls (baseball, tennis ball, golf ball, etc.).
2. Divide students into small groups (minimum of three).
3. Each group takes a piece of paper about 2 meters long from the paper roll. (The exact length is not important.)
4. Students use the tape measure and marker pen to mark the paper at 10-centimeter intervals along its length.
5. Each group should choose one ball to work with. (The activity will work if groups use the same kind of ball, but results will be more instructive if each group works with a different ball.)
6. Locate a suitable outdoor space, ideally with a light-colored wall or fence against which a thrown ball can be easily seen.
7. Groups use the duct tape to tape their paper to the wall. The paper should be oriented vertically to provide a scale on which to measure the height of the ball when it is thrown. (If the tape doesn't stick to the wall, a wooden or chain-link fence may work better.)
8. Students will need to estimate the initial velocity of the ball when it is thrown. If time allows, they can estimate velocity by throwing the ball vertically and using the time taken for the ball to rise and then fall (since change in velocity = acceleration due to gravity × half the time taken for the ball to return to its starting point). Alternatively, students can "guesstimate" using a velocity of 10 meters per second.
9. Groups conduct several trials to throw the ball in a parabola, while filming the path of the ball, as follows:
    i. One student is positioned to film the path of the ball, and begins to film.
    ii. Another student throws the ball upward at a steep angle (> 45° will work).

HOW A BALL FALLS

iii. As the ball is released, a third student starts the timer.
iv. When the ball hits the ground, the student stops the timer.

10. Students repeat this procedure for several trials. If time allows, five to ten trials will suffice, but the activity will work with just one or two trials.
11. Back in the classroom, groups work on modelling their activity using a quadratic equation. For example, if the initial velocity of the ball is 10 meters per second and it is released at a height of 1 meter, the height, h, of the ball at any time, t, is given by $h = 1 + 10t - 4.9t^2$. (The quadratic coefficient is simplified from the acceleration due to gravity = 9.8 m/s^2.)
12. They then use their data to create a model of the trajectory by estimating the values of t and h.
13. They factor the quadratic equation to determine values of t and h.
14. The teams use the values from factored equations to generate a graph that models the parabola.
15. Teams compare the shape of the parabola in their graph with the path of the ball they recorded on video. (If time allows, students could draw a representation of the ball's path from the video, and then map this to the graph from their model.) The aim is for students to qualitatively evaluate the fit of their model with the path of the ball they recorded.
16. Teams present their results for peer evaluation of the accuracy of their mathematical models compared with their video observations.

Take action!

- Contact your school sports or junior league coach. Explain the methods you used to model the parabola of a moving ball. Explore ways with the coach in which you could apply this method to improve the performances of ball sports players.

- Write a blog article about using quadratic equations to model everyday phenomena. Publish the article on your school blog or contact websites that may be interested in your article as a guest author.

HOW A BALL FALLS

NATIONAL STANDARDS

Science	HS-PS2-1 Motion and Stability: Forces and Interactions SEP: Developing and Using Models SEP: Analyzing and Interpreting Data DCI: PS2.A: Forces and Motion
Technology Education	HS-ETS1-2 Engineering Design International Technology and Engineering Educators Association 9-12: 10.P, 11.P, 12.L, 12.P, 17.L, 17.P
Mathematical Practice	CCSS.MATH.CONTENT.HSF.LE.A.1 CCSS.MATH.CONTENT.HSA.CED.A.1 CCSS.MATH.CONTENT.HSA.CED.A.2 MP.2 MP.4
English Language Arts	CCSS.ELA-LITERACY.RST.11-12.7

HOW A BALL FALLS STUDENT WORKSHEET

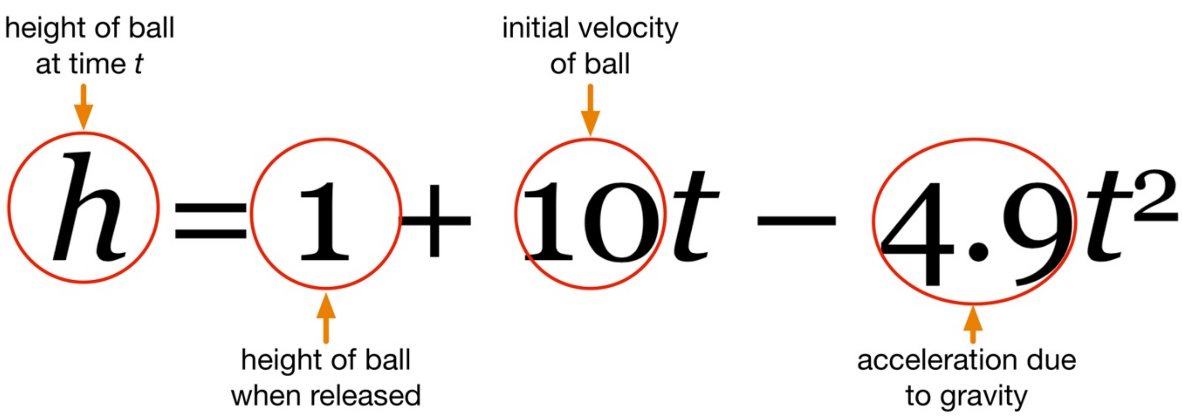
Background
When you toss a ball in the air, it rises and then falls to the ground. In this activity, you will use a quadratic equation to model the path of a thrown ball. You will make a video of a ball being thrown. You will then use the video to determine the path of the ball and formulate a quadratic equation that best fits the path of the ball.

Activity
You will work in small groups. Follow the instructions:

1. Cut a piece of paper about 2 meters long from the paper roll. (The exact length is not important.)
2. Use the tape measure and marker pen to mark the paper at 10-centimeter intervals along its length.
3. Choose a ball to work with. If possible, use a ball that is not being used by another team.
4. Locate a suitable outdoor space, ideally with a light-colored wall or fence against which a thrown ball can be easily seen.
5. Use the duct tape to tape their paper to the wall. Orient the paper vertically. The marked paper provides a scale on which to measure the height of the ball when it is thrown. (If the tape doesn't stick to the wall, a wooden or chain-link fence may work better.)
6. Estimate the initial velocity of the ball when it is thrown. If time allows, estimate velocity by throwing the ball vertically and using the time taken for the ball to rise and then fall (since change in velocity = acceleration due to gravity × half the time taken for the ball to return to its starting point). If time is short you can use a velocity of 10 meters per second.
7. Ready the camera and conduct several trials to throw the ball in a parabola, while filming the path of the ball, as follows:
    a. Position one student to video the path of the ball and start to video just before the ball is thrown.
    b. Another student throws the ball upward at a steep angle (> 45° will work), but not vertically.
    c. As the ball is released, a third student starts the timer.
    d. When the ball hits the ground, the student stops the timer.
    e. For each trial, record the height reached by the ball and the time taken between its release and hitting the ground.
8. Repeat step 7 for several trials. If time allows, five to ten trials will suffice, but the activity will work with just one or two trials.
9. Back in the classroom, use a quadratic equation to model the ball's path. For example, if the initial velocity of the ball is 10 meters per second and it is released at a height of 1 meter, the height, h, of the ball at any time, t, is given by $h = 1 + 10t - 4.9t^2$. (The quadratic coefficient is simplified from the acceleration due to gravity = 9.8 m/s^2.)
10. Estimate the values of t and h to create a mathematical model of the trajectory.
11. Factor the quadratic equation to determine the values of t and h.

12. Use the values from factored equations to generate a graph that models the parabola.
13. Compare the shape of the parabola in your graph with the path of the ball you recorded on video. (If time allows, draw a representation of the ball's path based on the video, and then map this to the graph from your model.) The aim is for you to qualitatively evaluate the fit of your model with the path of the ball you recorded.
14. Present your results for peer evaluation of the accuracy of your mathematical models compared with your video observations.

Model
Your model of the ball's path includes the terms shown in the diagram:



Substitute the values in the above terms with the values you obtain from your activity.

Results
Use the table to record the height reached by the ball and the time taken between its release and hitting the ground.

Trial	Height of ball when released	Maximum height of ball (*h*)	Time taken (*t*)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
Mean			

Analysis

1. Use the data from your results (step 6) to determine a value for the ball's initial velocity (the coefficient of the second term in the equation).
   Estimated value of ball's initial velocity: ______________
2. Use this value to calculate the values of *t* and *h* from the factored equation.
3. Use graph paper or graphing software to draw a parabola for your calculated values of *t* and *h*. (Note that when the ball hits the ground, $h = 0$.)
4. Answer the questions.

What is the mean height of the ball when released?

What is the mean of the ball's maximum height?

What is the mean time taken?

How well does your equation model the actual paths taken by the ball?

What factors and assumptions could cause differences between your model and the actual path you observe in the videos of the ball?